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Performance Measures of M/M/C/N Queueing Model

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ABSTRACT:
In this paper, we study steady-state parameters and performance measures of M /M / C / N queueing system. In this model capacity of the system is finite.

Keywords: Steady-state, performance measures, queueing system, finite capacity.

1. INTRODUCTION:

Queues are generally phenomena that happen all the time. Everyone has already joined a queue at least once when driving home and lining up in a traffic jam, when taking tickets, paying bills etc.
We consider a finite capacity model. In this system there are C servers and capacity of the system is finite N. Abou-El-Ata and Hariri [1992] discussed multi-server M / M / C / N queues with balking and reneging and they have derived steady state probabilities. F.S.Q. Alves et al. [2011] analysed upper bound on performance measures of Heterogeneous M / M / C queues. A queueing model of call centres was discussed by Koole, G. and Mandelbaum in [2002].

1.1 Notations and Assumptions

- Customers arriving to Poisson process with arrival rate λ. The service time is distributed according to an exponential distribution with service rate μ.
- If a customer who upon arrival will be serviced immediately, if some servers are busy and some servers are idle.
- The service order is assumed to be First Come First Served (FCFS) basis and interarrival times, service times and vacations are mutually independent.

2. RELATED WORK

W. Yue et al. [2009] discussed advances in queueing theory and network applications. The M/M/C/N queu with balking and reneging was analysed by M. Abou-El-Ata and A. Hariri in [1992]. T. Van Woensel and F.R.B. Cruz [2009] gave a stochastic approach to traffic congestion costs. Gans N, Koole, G and Mandelbaum [2003] studied telephone call centers, Tutorial review and research prospectus.
Jain, M., Singh M. and Baghel K.P.S. [2000] studied M/M/C/K/N/ Machine repair problem with balking, reneging additional repairman. Gautam Natarajan [2012] gave analysis of queues methods and application. A. Shawky [2000] performed the machine interference M/M/C/K/N model with balking, reneging and spares.

3. Performance Measures of M / M / C / N queueing model

The steady-state distributions are defined as follows:

$$\lambda_n = \begin{cases} \lambda & : \quad 0 \le n < N \\ 0 & : \quad \text{otherwise} \end{cases}$$

$$\mu_n = \begin{cases} n\mu & : \quad 0 \le n < C \\ c\mu & : \quad C \le n \le N. \end{cases}$$

The probability of n customers in the system

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0; & 0 \le n \le C \\ \frac{1}{C^{n-c}C!}\left(\frac{\lambda}{\mu}\right)^n P_0; & C \le n \le N \end{cases}$$

And Probability of no customer in the queue is given as follows:

$$P_0 = \begin{cases} \left[\sum_{n=0}^{c-1} \frac{1}{n!}r^n + \frac{1}{C!}(r)^c[1-\rho^{N-C+1}]\frac{c\mu}{c\mu-\lambda}\right]^{-1}; & \frac{\lambda}{c\mu} \neq 1 \\ \left[\sum_{n=0}^{c-1} \frac{1}{n!}r^n + \frac{1}{C!}r^c(N-C+1)\right]^{-1}; & \frac{\lambda}{c\mu} = 1 \end{cases}$$

where $\rho = \frac{\lambda}{c\mu}; \quad r = \frac{\lambda}{\mu}$

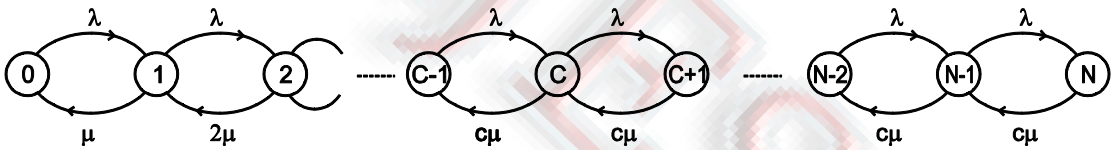

Figure: M / M / C / N Flow rate (When c > 1).

3.1 Measurements of the Model

(i) Avg. no. of customer in the queue

Or $$E(m) = \frac{P_0 r^c.\rho}{c!(1-\rho)^2}\left[1-\rho^{N-C+1}-(1-\rho)(N-C+1)\rho^{N-C}\right]$$

Where $\frac{\lambda}{c\mu} \neq 1$

(ii) Average number of customer in the system is

$$E(n) = \sum_{n=0}^{N} nP_n$$

$$= E(m) + c - P_0 \sum_{n=0}^{c-1} \frac{(c-n)(\rho c)^n}{n!}$$

(iii) Avg. waiting time in the system is given by

$$E(V) \text{ or } W_s = \frac{L_s}{\lambda(1-P_N)}$$

(iv) Avg. waiting time in the queue given as follows:

$$E(w) \text{ or } w_q = \frac{L_q}{\lambda(1-P_N)}$$

3.2 Special cases

If $N \to \infty$ i.e. capacity of the system is infinite then we have the following results.

1. $$\lim_{N\to\infty} P(waiting) = \begin{cases} \frac{1}{\rho}; & \text{if } \rho > 1 \\ 1; & \text{if } \rho = 1 \end{cases}$$

2. $$\lim_{N\to\infty} P(blocking) = \begin{cases} 1-\frac{1}{\rho}; & \rho \geq 1 \\ 0; & 0 < \rho < 1 \end{cases}$$

3. $$\lim_{N\to\infty} P(no-waiting) = \begin{cases} 0; & \rho \geq 1 \\ 1-r; & 0 < \rho \leq 1 \end{cases}$$

4. If N = C, then model M / M / C / N reduces to M / M / C / C model. Prob.(waiting) = 0.

Applications of Queueing theory

Queues may cause the quality of the services or the prices of the goods to rise or fall depending on the efficiency of the distribution and logistics. Organizing queuing system in order to decrease the erline length can be a way to reduce costs is maximizing profit.

In M / M / C / N model, if we consider N = C then model is M / M / C / C queueing model, also known as Erlang Loss system. This model is useful in call center. Call center is a department of an establishment that attends to customers via telephone conversation. Generally purpose of these centers are purpose of sales and product support (Another department also attends to e-mails, faxes; letters like similar written correspondence, that center is referred as contact center).

CONCLUSION

In this paper, we have discussed finite source model the M/M/C/N model and performance measures of this model. Also we have reviewed some special cases of this model.

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